PHYS 798C Spring 2024
Lecture 13 Summary

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Tunneling in to Superconductors

Tunneling into and out of a superconductor gives insights in to the density of states, the energy gap, as well as the pairing mechanism. Here we focus on **quasiparticle** tunneling into superconductors, as opposed ot Cooper pair tunneling between superconductors, which will be discussed later. A curious result is that coherence effects are largely suppressed in single-particle tunneling. We will also consider strong-coupling effects through the Eliashberg generalization of BCS theory, and see how strong-coupling effects influence the tunneling conductance at high bias.

A. Tunneling Hamiltonian

We imagine two metals separated by an insulating tunnel barrier. A potential difference is applied between the two metals and the resulting net current is measured. In other words, one measures the "I-V Curve" of the junction. We will also consider the differential conductance dI/dV of the tunnel junciton. Here we consider only single particle (as opposed to Cooper pair) tunneling. The tunneling Hamiltonian is

$H_T = \sum_{\sigma,k,q} T_{kq} c^+_{k\sigma} c_{q\sigma} + \sum_{\sigma,k,q} T^*_{qk} c^+_{q\sigma} c_{k\sigma}$,
where the first term is for forward tunneling and the second term is for reverse tunneling. Momenta k, q refer to the left and right metal, respectively. This assumes no spin-flip in the tunneling process.
Because the insulating barrier does not support quasiparticle states, to describe tunneling one has to extract an electron or hole from one metal, realizing the particle in some sense, and then deposit it in the other metal. This essentially destroys the coherence effects discussed in the last lecture, which arise from the fact that Bogoliubon's are a coherent superposition of electron and hole.
Josephson introduced new operators to create electrons and holes in the metals with probability unity. Josephson's electron and hole creation operators are,
$\gamma^+_{ek0} = u^*_k c_{k,\uparrow} - v^*_k S^+_k c_{-k\downarrow}$
$\gamma^+_{hk0} = u^*_k S_k c^*_{k,\uparrow} - v^*_k c_{-k\downarrow}$,
where S^+_k creates a $(k,\uparrow)$, $(-k,\downarrow)$ Cooper pair, and satisifes $S^+_k S_k = 1$.
In fact one can show that $\gamma^+_{hk0} = S_k \gamma^+_{ek0}$. These excitations still have energy $E_k = \sqrt{\xi^2_k + \Delta^2_k}$.

On page 72, Tinkham shows that coherence effects are lost when considering the tunneling Hamiltonian. This leads to some simplifications in understanding tunneling processes into or out of a superconductor.

B. Tunnel Current

Because coherence effects are lost, one can use a semiconductor model of (incoherent) single particle states for tunneling. The superconducting density of states is reflected about the chemical potential and all single-particle states below the Fermi energy are filled at zero temperature, and all single-particle states above are empty. At finite temperature the Fermi distribution $f(E)$ describes the number of quasiparticles in the excited states.

The net tunneling current is given by,
$I = A \int_{-\infty}^{+\infty} |T|^2 D_L(E) D_R(E+eV) \left[f(E) - f(E+eV)\right] dE$, where L refers to the left metal and R refers to the right metal, and V is the potential difference, T is some (assumed independent of energy) tunneling matrix element, and $D(E)$ is the density of states. Note that the energy integrals are interrupted in the range where the densities of states are zero.
The tunneling current is clearly dominated by the energy dependence of the densities of states $D(E)$ of the two banks.

There are three cases to consider.

C. N-I-N, N-I-S and S-I-S Tunneling

In the N-I-N case we take the DOS to be constant near the Fermi energy, and the I-V curve is linear in voltage: $I_{NIN} = G_{NN}V$ with $G_{NN} = A\left|T\right|^2 D_L(0)D_R(0)e$.

In the S-I-N case at zero temperature there will be no current until the Fermi energy of the normal metal lines up with the gap edge in the superconductor, i.e. $eV = \pm\Delta$, at which point the current quickly rises and then eventually increases linearly with voltage. At finite temperature the turn-on at Δ is washed out by the excited quasiparticles in the normal metal.
The differential conductance can be written as,
$G_{NS} = \frac{dI_{NS}}{dV} = G_{NN}\int_{-\infty}^{+\infty} \frac{D_S(E)}{D_N(E)}\left[-\frac{\partial f(E+eV)}{\partial(eV)}\right]dE$.
The $D_S(E)$ DOS term is strongly peaked at Δ and the Fermi function derivative becomes more and more strongly peaked as temperature approaches zero. In the limit of zero temperature it becomes a delta function and picks out the superconducting DOS at the voltage bias. As such the low-temperature differential conductance directly measure the superconducting DOS.

In the S-I-S case there will be current peaks at voltages that line up the gap edges for both $eV = \Delta_L + \Delta_R$ and for $eV = |\Delta_L - \Delta_R|$.
One can use these gap features in the $I - V$ and dI/dV curves to map out the gap $\Delta(T)$ as a function of temperature, as shown for the case of Al here.

D. Strong Coupling Superconductors and Eliashberg Theory

The BCS weak-coupling approximation $D(E_F)V << 1$ gives universal results for the gap ratio $\Delta(0)/k_BT_c$, specific heat jump at T_c, isotope effect exponent, etc. However many superconductors show deviations from these universal values, and often tend to systematically deviate on one side of the universal value (e.g. they almost always show $\Delta(0)/k_BT_c > 1.76$). The over-simplified treatment of the pairing interaction is partly responsible for these deviations. Eliashberg developed a more complete treatment of the pairing interaction by introducing the function $\alpha^2(\omega)F(\omega)$, where $\hbar\omega$ is the energy of the Bosons that provide the electron pairing, $\alpha(\omega)$ is the electron-Boson coupling strength, and $F(\omega)$ is the Boson density of states. For superconductors paired by the electron-phonon interaction, substitute the word 'phonon' for Boson.

Strong coupling means that the phonon spectrum is modified by interactions with the electrons, and the electron energies are modified by their interactions with the phonons. These mutual interactions have to be resolved self-consistently. Basically, the single particle states labeled by k, σ are no longer good eigenstates in the presence of strong electron-phonon coupling. The class web site for this lecture shows expressions for the moment λ of the $\alpha^2(\omega)F(\omega)$ distribution, which roughly replaces the phenomenological factor V in the Cooper pairing potential. The theory also includes the Coulomb repulsion between the electrons through a coefficient μ^*. The theory successfully explains the deviations from BCS universal values. More importantly, tunneling data at high bias (greater than Δ) reveals the finite lifetime of the quasiparticles as they scatter, due to phonon interactions. It is possible to invert the tunneling conductance vs. bias data to extract the $\alpha^2(\omega)F(\omega)$ function. In the case of Pb this was found to closely resemble the phonon density of states. This provides strong evidence that the Boson that produces Cooper pairing is the phonon, at least in Pb, and many other 'conventional' superconductors.

In more detail, the gap becomes complex, and a function of energy $\Delta(E)$. The energy-dependent phase of this gap is distinct from the coherent phase of the BCS gap. The imaginary part of the gap, $Im[\Delta]$, is due to the decay of the quasiparticles and the creation of real phonons. The real part of the gap, $Re[\Delta]$, goes through a resonant absorption when $E \approx \hbar\omega_{phonon}$. This dispersion of the complex gap is evident on slide 9 of the slides posted on the class web site.